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Light transmission through two slits: the Young experiment revisited

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Abstract
In the traditional analysis of Young interference it is supposed that the two slits radiate in the same way for any distance between them. In this work, the phenomenon of interference is studied within the framework of the classical electromagnetic theory and in a context that takes into account the mutual interactions between the slits. The results obtained are in agreement with recent experimental values showing how the total transmitted power is enhanced or reduced as a function of the distance between the two slits.

Keywords: interference transmission, two-slit interactions

(Some figures in this article are in colour only in the electronic version)

1. Introduction

It is well known that due to the interference phenomenon the radiation emitted by two identical sources (with the same power, frequency and phase) is in some directions four times greater than that produced from only one of the sources acting alone. This fact is explained (French 1971) by arguing that the increase by a factor of two in some directions is compensated by the existence of zero intensity in others. In this argument is implicitly the idea that interference is basically a distribution of power in space, and that the total emitted power is the sum of the power radiated by the two sources separately.

In the traditional analysis of the radiation diffracted by two slits it is supposed that the field distribution over them is originated only by the incident monochromatic beam of light. However, the fields over the slits mutually interact and their intensities may be increased or reduced by this interaction.

The mutual interaction between the slits may be very important when the distance between them is of the same order as or less than the wavelength. In most of the arrays used to study the interference of electromagnetic waves in the visible range, the distance between sources is much greater than the wavelength. Then, the effect of the mutual interaction on the total radiated power of the two slits is not observed. This could be the reason why most of the textbooks treating the interference of light waves completely ignore the mutual interaction between sources.

In the frequency range of radio waves, the mutual interaction between antennae is a well known phenomenon. In this case, the mutual interaction between antennae is described through the so-called mutual impedances (Collin 1985). In general, the computation of these mutual impedances is very complex because the knowledge of the fields in the near zone is required.

The mutual interaction between emitters of sound waves has been analysed quantitatively by Scandrett *et al* (2001), using a method suggested by Pritchard (1960) that requires only the knowledge of the field in the far zone. This reduces the complexity of the involved calculations. To our knowledge, this 'acoustic' method has not been used so far for the computation of mutual impedances between antennae.

In recent years, experimental results on the interference of light passing through an array of small holes perforated in gold films have been obtained (Ghaemi *et al* 1998, Sonnichsen *et al* 2000). The distance between holes is comparable to the wavelength employed. In these experiments, some apparent anomalies were observed in the light intensity transmitted by the holes that could be explained if the mutual interactions were taken into account.

In this work, the Pritchard procedure is adapted to study the interference of light waves diffracted by two slits. The results obtained are in agreement with recent experiments (Schouten *et al* 2005) that show a strong modulation of the total transmitted power as a function of both the incident

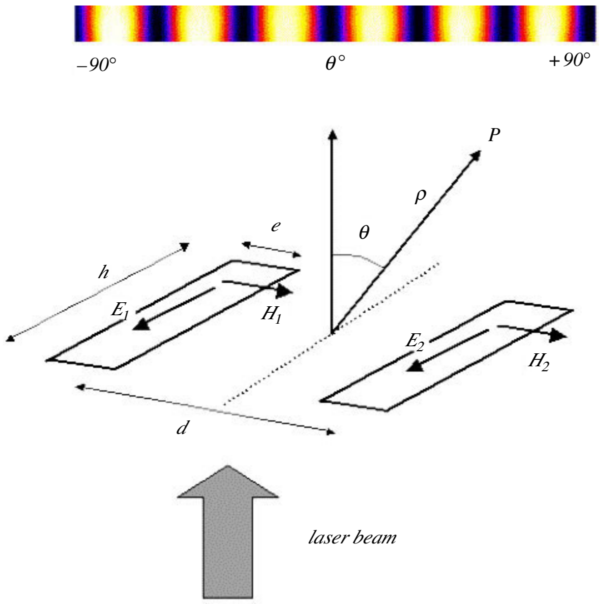

Figure 1. An 'infinite' absorbent screen with two parallel slits of length h, width e, separated by a distance d, is shown. The frame at the top shows the Young type interference pattern behind the screen.

wavelength and the distances between slits. Therefore, as has been suggested by Welti (2002), the interference of the diffracted light by the two slits has a behaviour similar to the interference of radio waves created by two antennae as well as to the interference of sound waves originated by two speakers.

2. The power transmitted through the two slits

In figure 1, an 'infinite' absorbent screen with two parallel slits of length h and width e, separated by a distance d, is shown. The slits are illuminated by a monochromatic plane wave of wavelength λ incident in the normal direction of the screen. We assume that $e \leqslant \lambda$ and $h \gg \lambda$.

If the electric field of the incident wave is parallel to the slits, the fields (E_1, H_1) and (E_2, H_2) over the slits have the directions shown in figure 1. If $d \gg \lambda$, the mutual interaction between slits is negligible. Consequently, the field over each slit may be approximated by the incident field.

In this situation, the electric and magnetic fields over the slits are related by the following equations:

$$\begin{aligned} E_1 &= Z_{11} H_1 \\ E_2 &= Z_{22} H_2 \end{aligned} \qquad (1)$$

where

$$Z_{11} = Z_{22} = Z_0 = \sqrt{\mu_0/\varepsilon_0}, \qquad (2)$$

is the intrinsic impedance of free space.

If the fields over the slits interact mutually, the electric and magnetic fields must be correlated by the following equations:

$$\begin{aligned} E_1 &= Z_{11} H_1 + Z_{12} H_2 \\ E_2 &= Z_{21} H_1 + Z_{22} H_2 \end{aligned} \qquad (3)$$

where Z_{12} and Z_{21} are coefficients that take into account the mutual interactions between the slits. Equations (3) establish that the magnetic field H_2 from slit 2 creates an electric field $Z_{12}H_2$ in slit 1, and that the magnetic field H_1 from slit 1 creates an electric field E_2 in slit 2. If both slits are identical, then $Z_{12} = Z_{21}$.

If the incident beam over the two slits is homogeneous, then $H_1 = H_2$, and $E_1 = E_2$. Therefore,

$$E_1 = E_2 = (Z_{11} + Z_{12}) H_1. \qquad (4)$$

The average power density over slits 1 and 2 is given by the real part of the complex Poynting vector,

$$S_1 = S_2 = \mathrm{Re}\left(\frac{1}{2} E_1 H_1^*\right) = \frac{1}{2} |H_1|^2 R_{11} \left(1 + \frac{R_{12}}{R_{11}}\right) \qquad (5)$$

where $R_{11} = Z_0$ and R_{12} is the real part of Z_{12}.

If we integrate the quantity given by (5) over the surface of two slits, the power W transmitted through the two slits is obtained as

$$W = 2W_0 \left(1 + \frac{R_{12}}{R_{11}}\right) \qquad (6)$$

where W_0 is the power passing through either slit in the absence of the other one. Due to the mutual interaction, we observe that the transmitted power W through the two slits is not the sum of the power of each slit acting separately.

To calculate W through equation (6), the knowledge of the 'mutual interaction coefficient' Z_{12} is required or, at least, its real part R_{12}. The computation of Z_{12} is very complicated as it requires the description of the electromagnetic fields in the zone close to the slits and the interaction mechanism between them.

3. Transmission coefficient of two slits

The total transmitted power W may be also obtained by calculating the Poynting vector flux in the far zone. Inserting this value in (6), we can compute the real part of the mutual interaction coefficient R_{12}. The intensity (the Poynting vector) of the diffracted wave by the two slits (see figure 1) in the far zone, at a point P placed at a distance ρ, is given by (Rossi 1967)

$$I(\theta) = 4 I_1(\theta) \cos^2\left(\frac{kd}{2} \cos\theta\right) \qquad (7)$$

where

$$I_1(\theta) = I_0\, D(\theta)$$

is the intensity of the emitted radiation by each slit when acting separately whereas I_0 is the intensity created at P by an isotropic radiator irradiating the same power as either source separately. The function $D(\theta)$ is the *directivity function* of each of the slits. If the slits are isotropic, then $D(\theta) = 1$.

The average total power W radiated by the system of two slits is obtained by integrating $I(\theta)$ given by (7) through a semicylindrical surface S of ratio ρ and height h, and with its axis on the middle line passing through the two slits (see figure 1).

$$W = \int_S 4 I_0 D \cos^2\left(\frac{kd}{2} \sin\theta\right) h\rho\, \mathrm{d}\theta. \qquad (8)$$

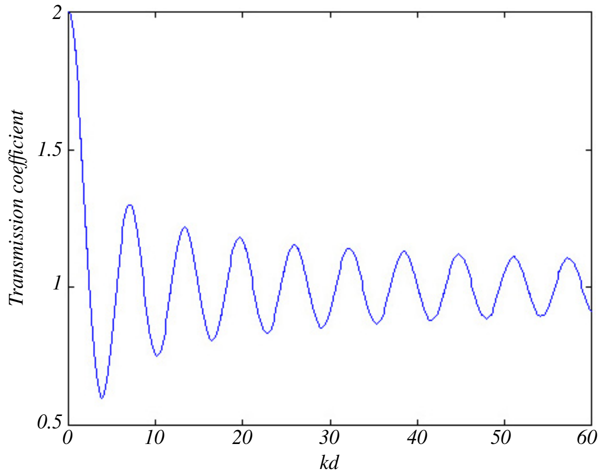

Figure 2. The calculated angular-integrated transmission coefficient of a double slit as a function of $kd = 2\pi d/\lambda$.

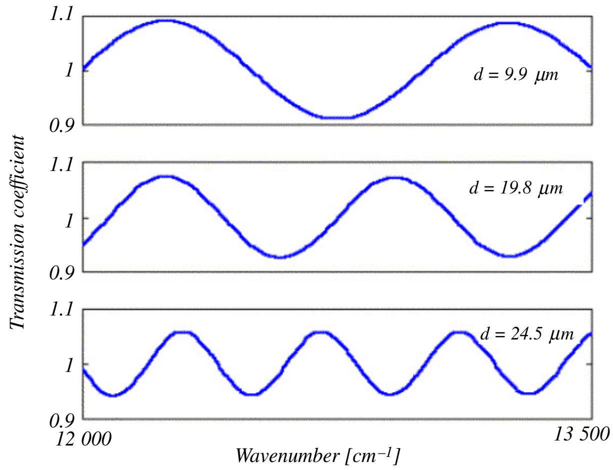

Figure 3. The calculated transmission coefficient as a function of the wavenumber that corresponds to the experience of Schouten *et al*. The value of the slit separation d is indicated in each frame.

By using the trigonometrical identity

$$\cos^2(x/2) = \frac{1+\cos x}{2},$$

equation (8) may be rewritten as

$$W = 2W_0\left(1 + \frac{1}{W_0}\int_S I_0 D\cos(kd\sin\theta)h\rho\,\mathrm{d}\theta\right),$$

where

$$W_0 = \int_S I_0 D h\rho\,\mathrm{d}\theta$$

is the power that would irradiate either source when the other one is not present.

If $e < \lambda$, the function $D \approx 1$ for any θ value, and consequently,

$$I_0 \approx \frac{W_0}{\pi\rho h},$$

then,

$$W = 2W_0\left(1 + \frac{1}{\pi}\int_{-\pi/2}^{\pi/2}\cos(kd\sin\theta)\,\mathrm{d}\theta\right).$$

Remembering that

$$\int_{-\pi/2}^{\pi/2}\cos(kd\sin\theta)\,\mathrm{d}\theta = \int_0^{\pi}\cos(kd\sin\theta)\,\mathrm{d}\theta = \pi J_0(kd),$$

where J_0 is the Bessel function of the first kind and zero order, we have the following result:

$$\tau = \frac{W}{2W_0} = (1 + J_0(kd)) \tag{9}$$

where τ is the *transmission coefficient* of the two slits. From (9) and (6), we obtain

$$R_{12} = \sqrt{\frac{\mu_0}{\varepsilon_0}}J_0(kd). \tag{10}$$

In figure 2, τ is shown as a function of $kd = 2\pi d/\lambda$. We observe that as $kd \to 0$, $J_0 \to 1$, and the transmission coefficient is twice the optical transmission coefficient through the two slits. As $kd \to \infty$, $J_0 \to 0$, and the transmission coefficient tends to unity, i.e., the total transmitted power is the sum of the transmitted powers through each one of the slits acting alone. Between these two limit values, the transmission coefficient has a periodic behaviour with an amplitude that decreases as the distance between slits is increased.

4. Comparison with experimental results

Schouten *et al* (2005) published experimental results on the optical transmission through a thin metal screen with two slits having a width less than a wavelength and separated by several wavelengths. They found that the total transmitted power is reduced or enhanced as a function of the wavelength of the incident light beam. They attribute this modulation to a phenomenon of interaction between the slits due to surface plasmon excitation propagating from one slit to the other.

The results obtained in this work, in which no hypothesis is made on the physical mechanism of the interaction between slits, are in satisfactory agreement with the measurements by Schouten *et al*. They illuminated the slits with a laser of variable wavelength λ between 0.740 and 0.830 μm. In figure 3, the normalized transmission coefficient $W/2W_0$ resulting from (9) is shown as a function of the wavenumber $(1/\lambda)$ for three distances between slits (d = 9.9; 19.8 and 24.5 μm).

We observe that the periodicity as well as the modulation of the transmission coefficients of figure 3 are in agreement with the experimental results by Schouten *et al* that are shown in figure 1 of their work.

As $kd \gg 1$, we can use the asymptotic form of the Bessel function J_0 to rewrite (9) as

$$W = 2W_0\left(1 + \sqrt{\frac{2}{\pi kd}}\cos\left(kd - \frac{\pi}{4}\right)\right). \tag{11}$$

Schouten *et al* deduce that the total power registered by their detector (W in our work) is proportional to twice the power radiated by each slit separately ($2W_0$) and that the modulation of the transmitted power may be approximated by the following expression:

$$W = 2W_0(1 + \beta^2 + 2\beta \cos(kd + \Phi)) \qquad (12)$$

where k is the propagation constant. They found that if $\beta \approx 0.1$, then the expression given by (12) fits adequately their measurements. Comparing (12) with (11), we find that

$$\beta = \sqrt{\frac{1}{2\pi kd}}. \qquad (13)$$

This β value depends on both the propagation constant k and the separation between slits. The β values that are calculated by means of (13) vary between 0.07 and 0.03 for the values of k and d in the experience of Schouten *et al*. Therefore, the modulation that they observed is deeper than the one that is calculated with (11). This difference may be attributed to the fact that in this work it is assumed that the slits are perforated on a 'neutral' absorbent screen, whereas the experiments are performed with a metallic screen that can enhance the mutual interaction between slits.

The modulation of the transmission coefficient that is obtained in this work is independent of the polarization of the incident light. However, Schouten *et al* observe this modulation only if the electric field of the incident light is perpendicular to the slits. This difference may be also be attributed to the assumption that the screen is absorbent and not a conductor.

Finally, we remark that small variations of d produce large variations in kd. In fact, d variations of the order or less than 0.1 μm produce variations of kd of the order of or greater than 1. Therefore, small experimental errors in the determination of d (and/or k) may lead to a shift of the maxima and minima shown in figures 2 and 3.

5. Conclusions

In this work, the phenomenon of interference is studied in a context that takes into account the mutual interactions between the sources. The results obtained are in agreement with recent experiments that show a strong modulation of the total transmitted power as a function of both the incident wavelength and the distance between sources. The physics of this interaction is very different according to the particular case considered: in aerials, the current from one of them creates an electric field, and this field induces a electro-motive force in the other; in the speakers, the motion of the membrane of one of them creates a pressure variation and this in turn produces a force on the membrane of the other. In the slits perforated on the surface of a thin metal layer, the physical mechanism is attributed to the surface plasmons. As a consequence of this interaction the 'amplitude of oscillation of the sources' is increased or reduced as a function of the relation among the distance between the sources and the wavelength. This explains the observed modulation of the emitted total power.

The fact to be remarked upon is that the interaction between the two sources, which is produced through physical processes that occur in the proximities of both of them, may be evaluated by means of the radiation fields in the far region. The interference phenomenon is formally the same for all kinds of waves. The mathematical expressions that describe it depend only on the geometry, the wavelength, the relative orientation, and the radiation diagram of both sources. They do not depend on the kind of wave that is being emitted. Therefore, the mutual interaction between sources has a certain 'universal' character.

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